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AMG-NYU Memo. No. 17
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DESCRIPTIVE SUMMARY OF THE THEORY OF SOUND SCATTERING BY RODS, RINGS, GRIDS AND SHIELDS UNDERWATER

By J. K. L. MacDonald

1. Introduction. The Applied Mathematics Panel Memo No. 20·1, by Professors Courant, Friedrichs and Stoker, developed a limit theory for the scattering of sound waves by thin shells of solid material underwater. Such shells are used in the form of streamlined "domes" enclosing anti-submarine sound projection and detection devices. Other shapes of solids, to which the above mentioned theory does not directly apply, are also used in these domes. In particular: reinforcing grids and ribs are often spot-welded to the inside surface of the shell, and large rigid shields are usually employed to reduce interference due, for example, to propeller noises. It is of interest to determine the distortion of sound beams due to such reinforcements and to determine the efficiency of shields. The present investigation was undertaken to develop formulas for such effects. A mathematical treatment of the problems is being prepared for separate distribution.

2. Main Results. It was found that the scattering properties of thin rods and grids are quite different from those of thin shells. In particular, for a commonly used type of grid the scattering was found to be nearly equivalent to that due to a shell of mass only one eighth that of the grid.

General formulas were obtained for the scattering due to individual rods, rings and shields. These formulas can be

used to calculate unusually large scattering intensities associated with accidentally bad spacing of scattering elements. Resonance effects for directive beams generated by vibrating areas on a sphere concentric with a dome were determined. Small changes in the radii or in the frequency were shown to produce marked changes in the total amount of scattering by the dome (and by the surface of the projector).

From the viewpoint of theory it is thought by the writer that the main feature of the work is the development of flexible equations for the treatment of general problems in scattering by use of properties which may be found in explicitly solvable cases.

3. Outline of Methods. Using the same hydrodynamical basis as in the memo mentioned in §1, adaptable forms of integral equations for the pressure amplitudes due to sound waves are obtained. These equations can be interpreted in a standard way as expressions for the amplitude at any point in terms of strengths of secondary sources on and in the scattering body. These strengths depend only on the local behavior of the amplitudes, and when the latter are known at least approximately, the scattering can be determined at other points. In practice the intensity of scattering observed at large distances from the projector is usually of major interest in connection with distortion of the beam of sound.

To determine the general orders of magnitude of the secondary sources, several known solvable cases were examined:

(1) Plane waves incident upon an infinite plane slab.

(2) Spherical waves in concentric shells.

(3) Plane waves incident upon a small sphere or upon a thin infinite cylinder.

(4) Plane waves upon a perfectly reflecting half plane.

Closed formulas were available for cases (1), (2) and (4), and expressions for (3) were derived suitable for calculations accurate within 2%. For small thicknesses the formulas for (1) and (2) reduce to those for thin shells developed in the Memo referred to above. A rather unexpected difference between the characteristics of thin shells and of thin rods or lumps was found. If the intensity of vibration of the projector is maintained, then the introduction of a thin shell hardly alters the amplitude in the water beside the shell, but inside the shell the rate of variation of amplitude with distance from the face is magnified in the ratio of "shell density to water density". For thin cylinders (or small spheres, or in general for thin rods and lumps) the reverse is the case (the pressure amplitudes being hardly altered in the region occupied by the cylinder, but magnification of the rate of change of pressure with distance outward near the cylinder being produced in the ratio of "solid density to water density".)

For a perfectly reflecting half plane the amplitudes are unaltered at the edge when the reflector is introduced. However, within about three wavelengths from the edge the amplitudes are almost doubled on the "exposed" side of the reflector, and are reduced almost to zero on the "shaded side". Since the

wavelength of sound used is about 6 cm as contrasted with around 0.00006 cm for light, acoustic shadow effects are relatively diffuse.

The different characteristic changes in the local amplitudes when thin shells, thin rods and large reflecting shields are placed in the path of sound waves correspond to the production of different characteristic secondary source distributions. These distributions, when known, permit the calculation of the different types of scattering produced by the respective forms of scattering bodies. In most of these calculations, various forms of Bessel functions appear, and, except for the case of large shields, the expressions are not very complicated.

A series expansion and some integral transformations permit the determination of "equivalent thin shells" for grids with bar thickness not more than one fifteenth of a wavelength, and with bar spacing not more than one half of a wavelength. It was found that the equivalent shell should have a mass equal to that of the amount of water displaced by the grid, provided that the amount of scattering is required only for distant points.

The shielding effect of a spherical surface of the projector on the waves scattered by the dome is described mathematically in terms of symmetry properties of Legendre functions and by use of asymptotic forms for Bessel functions. Explicit "acoustic images" ~~in perfectly absorbing~~ and in perfectly absorbing and in perfectly reflecting spheres are obtained. General shielding effects are still under investigation.

The writer wishes to express his indebtedness to Professor Courant for suggesting and supporting the investigation, and to Professors Courant, Friedrichs and Stoker for helpful comments and kind encouragement.

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